Path Homology Theory of Digraphs and Its Applications to Brain Network Analysis

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Abstract

Traditional graph theoretical analysis of brain networks rests on the simple premise of pairwise relationships, that is, nodes connected through edges, but this framework fails to capture the higher-order organization of neural systems. We present a framework based on the *path homology* of digraphs, introduced by Grigor'yan, Lin, Muranov, and Yau. Unlike clique-based descriptors, path homology detects topological features invisible to simplicial methods, such as double edges and directed squares. After reviewing elementary p-paths, the boundary operator, the chain complex of ∂-invariant paths, Betti numbers, and Euler characteristics, we turn to brain connectivity inference. Effective connectivity is inferred using information Partial Directed Coherence (iPDC), a frequency domain, multivariate estimator, based on the concept of Granger causality, obtained from an MVAR model. We report simulation studies computing Betti numbers and Euler characteristics across four random digraph models (k-regular, Erdős–Rényi, Watts–Strogatz, and Barabási–Albert). Finally, we apply the framework to scalp EEG recordings from patients with left temporal lobe epilepsy (Siena Scalp EEG Database), estimating dynamic iPDC networks across pre-ictal, ictal, and post-ictal phases and computing their path homologies. The results suggest that path homology captures higher-order network reorganization during the seizure.

Keywords: Path Complexes; Path Homologies; Information Partial Directed Coherence; Brain Connectivity Networks; Epilepsy.

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